Ternarylogic

Title: What if one plus one is not two, but seven?

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Date: July 14, 2005

Suppose 1+1 is not 2

Most people probably at one time have been involved in a discussion wherein somebody brings up the point that the arithmetical expression 1+1=2 is just a convention. And that it is possible to create a valid expression wherein for instance 1+1=7.

The question that arises is: is that possible? And are there rules that have to be applied?

First of all it is assumed that the statement $1+1\neq 2$ is not merely one of naming, in the sense that 2 in a new naming convention actually means something else, like 3 and that 7 actually means 2.

Secondly. The statement 1+1 is a not counting statement. Counting statement means: I count a first object and then I count another object. In that case I have counted two objects and so 1 + 1= 2.

Thirdly. The statement does not pertain to a binary or radix-2 arithmetic. There is no 2 in a radix-2 arithmetic, unless the naming convention is changed. In radix-2: 1 +1 = 1 0.

So what if $1 + 1 \neq 2$

In general the statement a + b = c, wherein a, b and c are single digits signifies the determination of the residue in a modulo-n addition. In a modulo-10 addition 1+1=2 and 2+3=5. When the sum is greater than 9, 10 is subtracted and the difference is the residue. So the residue of 7+9 is 6. That is 16-10. Determining the residues in an addition is one operation of determining the sum.

The second operation that is required for establishing a sum is the determination of the carry digit. This operation, in standard radix-10 addition, can either be 0 or 1. When the addition of two digits is a sum greater than 9 the Carry is a 1. When the sum is 9 or less the Carry is 0.

In this example arithmetic and logic are mixed. It is possible to describe the addition as a 10-valued logic operation. The 10-valued logic function '+' and the function C are used.

The following two truth tables (or switching tables) describe the two logic functions '+' and C.

Function ‘+’:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

The function ‘+’ is the modulo-10 addition.

The function C:

C	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	1
4	0	0	0	0	0	0	1	1	1	1
5	0	0	0	0	0	1	1	1	1	1
6	0	0	0	0	1	1	1	1	1	1
7	0	0	0	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1

The function C determines when a carry will be generated as a result of adding two digits. The function ‘+’ will be renamed as ‘R’.

The following table provides the different expressions to determine the digits of the sum of a radix-10 addition of two 2-digit numbers ab and cd.

					-	a	b
					-	c	d
residue					-	(aRc)	(bRd)
carry					-	(bCd)	-
residue					-	(aRc)R(bCd)	-
carry					(aRc)C(bCd)	-	-
residue					(aRc)C(bCd)	(aRc)R(bCd)	(bRd)

The expressions in the bottom row of the table show the individual digits of the sum. When numbers with more digits are added the expressions will expand. The expressions have to be evaluated until a row does not contain any valid Carry digits. The result will appear in the last ‘residue’ row. This approach is of course the well known ripple adder.

The functions ‘+’ and C may be called primitive functions. This means that for adding two single digit numbers only ‘+’ and C can be used to achieve the correct result. However it is clear that adding two digit numbers (or more-digit numbers) will create extended or composite expressions as a result of the carry ripple effect.

Composite expressions in many cases can be replaced by equivalent expressions, applying the same number of functions, wherein some or all of the functions are not primitive.

In order to create the equivalent expressions the primitive expressions have to be completed by considering the first carry occurring in the addition of the last two digits. It is automatically assumed that this carry is 0. Using a more formal approach it can be said that this carry is 0 because the digits on the decimal positions are actually 0. It is also important to be aware that leading and trailing 0s are dropped from the numbers.

					0	a	b	0
					0	c	d	0
residue					0	(aRc)	(bRd)	
carry					0	(bCd)	(0C0)	
residue					0	(aRc)R(bCd)	0	
carry					(aRc)C(bCd)	0	0	
residue					(aRc)C(bCd)	(aRc)R(bCd)	(bRd)	

A 10-valued equivalent expression

The following truth table shows a function R that can be applied in an equivalent composite expression.

R	0	1	2	3	4	5	6	7	8	9
0	9	8	7	6	5	4	3	2	1	0
1	8	7	6	5	4	3	2	1	0	9
2	7	6	5	4	3	2	1	0	9	8
3	6	5	4	3	2	1	0	9	8	7
4	5	4	3	2	1	0	9	8	7	6
5	4	3	2	1	0	9	8	7	6	5
6	3	2	1	0	9	8	7	6	5	4
7	2	1	0	9	8	7	6	5	4	3
8	1	0	9	8	7	6	5	4	3	2
9	0	9	8	7	6	5	4	3	2	1

The expression (a + b) using this function will be (a R b). By using this function the expression (1 + 1) will generate (1 + 1) = 7.

The alternative function C in the equivalent expression has the following truth table:

C	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	9
2	0	0	0	0	0	0	0	0	9	9
3	0	0	0	0	0	0	0	9	9	9
4	0	0	0	0	0	0	9	9	9	9
5	0	0	0	0	0	9	9	9	9	9
6	0	0	0	0	9	9	9	9	9	9
7	0	0	0	9	9	9	9	9	9	9
8	0	0	9	9	9	9	9	9	9	9
9	0	9	9	9	9	9	9	9	9	9

The addition of two 2-digit numbers will then look like:

Number 1 **ac**:	0	5	2
Number 2 **cd**:	0	8	9
residue	9	6	8
carry	9	9	0
residue	1	4	1

Though the intermediate steps look different from the well known modulo-10 addition and carry generation, the complete equivalent expression generates the correct answer.

This is a relatively simple example of possible equivalent expressions. But is shows what could and should happen if (1 + 1) = 7.

About Ternarylogic LLC: Ternarylogic LLC is a New Jersey based R&D company. Its mission is to create novel MVL technology. The company owns a portfolio of inventions related to scramblers/descramblers, sequence generators and sequence detectors, sequence correlators, gates and inverters based circuitry, non-binary multipliers, latches and other non-volatile memory elements, optical disk applications and MC-DSSS technology.