Ternarylogic

Title: N-valued inverters

Author: Peter Lablans, Ternarylogic LLC.

Date: May 10, 2007

The inverter in binary logic

An inverter in binary switching changes the state of a signal into its opposite. Usually one uses the states 0 and 1 in binary technology. That does not mean of course that the signals representing those states should be for instance 0 or 1 Volts. It is not even required that a 0 is represented by an absence of signal, though that is usually the case.

The notation of an inverter used herein is [x y]. The states x and y are from {0,1}. Each element in inverter [x y] has a state and a position. Using position with origin 0 the state x has position 0 and the state y has position 1. The position 0 represents an incoming signal in state 0 and position 1 represents a signal in state 1.

Accordingly one may read inverter [x y] as: [x y] is an inverter that changes an incoming signal in state 0 into an outgoing signal in state x; and it changes an incoming signal in state 1 into an outgoing signal in state y.

One may put this as: $\begin{bmatrix}0\\1\end{bmatrix}\rightarrow\begin{bmatrix}x\\y\end{bmatrix}$.

In general one accepts a binary inverter to change a state into its inverted or opposite state. That means that the binary inverter can be written as:

$$\begin{bmatrix}0\\1\end{bmatrix}\rightarrow\begin{bmatrix}1\\0\end{bmatrix}.$$

In binary logic there is just one inverter that changes a state into another state. That is of course because in binary logic there is just one other state.

The inverter will be provided with a broader meaning herein. An inverter will be defined as a transformation of an incoming state into an outgoing state. Using that definition one then has in binary logic 4 inverters: [0 0], [0 1]; [1 0] and [1 1]. In the extended notation one can write the 4 binary inverters as:

$$\begin{bmatrix}0\\1\end{bmatrix}\rightarrow\begin{bmatrix}0\\0\end{bmatrix};\begin{bmatrix}0\\1\end{bmatrix}\rightarrow\begin{bmatrix}0\\1\end{bmatrix};\begin{bmatrix}0\\1\end{bmatrix}\rightarrow\begin{bmatrix}1\\0\end{bmatrix}; \textit{and}\begin{bmatrix}0\\1\end{bmatrix}\rightarrow\begin{bmatrix}1\\1\end{bmatrix}.$$

The inverter [0 0] changes all incoming signals to state 0. If state 0 is absence of signal then [0 0] is an interrupted connection.

The inverter [0 1] passes incoming states as they are. If state 0 is absence of signal then [0 1] is an uninterrupted connection such as a piece of wire.

The inverter [1 0] changes an incoming state to its opposite state.

The inverter [1 1] always provides a state 1. As such it could be the output of a signal source that is always on.

The inverter [1 0] is the only reversible inverter. This means that [1 0] has a counterpart that reverses what the inverter achieved. The counterpart is of course the reversible inverter [1 0] itself. Applying [1 0] twice is like applying [0 1]. The inverter with a self reversing property is called a self reversing inverter.

N-valued inverters

The Ternary Case

As an example ternary inverters will be introduced. A ternary inverter is written as [x y z]. Herein x, y and z are one of {0, 1, 2}. It should again be pointed out that 0, 1 and 3 are states with the only meaning that each state is different from another state. The states in [x y z] have again a position starting with 0. So x has position 0; y has position 1 and z has position 2. The notation [x y z] may be considered shorthand for the transformation:

$$\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

There are of course 3*3*3 = 27 possible ternary or 3-valued inverters ranging from [0 0 0], [0 0 1], [0 0 2], [0 1 0], …, etc to [2 2 2].

Of special interest are the reversible ternary inverters. There are 3! = 6 reversible inverters. These inverters are [0 1 2]; [0 2 1]; [1 0 2]; [1 2 0]; [2 0 1] and [2 1 0]. The first one [0 1 2] is an identity inverter; the result of transformation of a state by this inverter is the same state. The second inverter [0 2 1] is a self reversing inverter. Applying this inverter twice will create identity. This can be shown in:

$$\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \underset{[0\,2\,1]}{\rightarrow} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}; \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \underset{[0\,2\,1]}{\rightarrow} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

The above should be read as: a transformation [0 2 1] is applied. Remember: a transformation is a change of one state into another state. As shorthand in [0 2 1] the state of the incoming signal is implied by the position of a state.

In column form the transformation shows a change of a state in one position to a state in a like position. The first transformation shows the inverter transformation. The second transformation may be confusing. But is shows the transformation of state 0 to state 0; of state 2 to state 1 and of state 1 to state 2 by applying inverter [0 2 1]. Accordingly it shows that applying [0 2 1] twice to a state results into the same state.

Inverter [2 1 0] is another self reversing inverter and so is [1 0 2]. The inverters [1 2 0] and [2 0 1] are also reversible inverters. They are called universal inverters. Each of the inverters can be realized by applying one of them at least once. Further more applying each inverter at least two times will create identity. This is shown in the following:

$$\begin{bmatrix}0\\1\\2\end{bmatrix}\underset{[201]}{\rightarrow}\begin{bmatrix}2\\0\\1\end{bmatrix};\begin{bmatrix}2\\0\\1\end{bmatrix}\underset{[201]}{\rightarrow}\begin{bmatrix}1\\2\\0\end{bmatrix};\begin{bmatrix}1\\2\\0\end{bmatrix}\underset{[201]}{\rightarrow}\begin{bmatrix}0\\1\\2\end{bmatrix}.$$

Applying [2 0 1] twice is like applying [1 2 0] once. Applying [2 0 1] three times is like applying the identity inverter.

The reversing inverter of inverter [2 0 1] is inverter [1 2 0], as applying first [2 0 1] followed by applying [1 2 0] will create identity. This is shown in the following:

$$\begin{bmatrix}0\\1\\2\end{bmatrix}\underset{[201]}{\rightarrow}\begin{bmatrix}2\\0\\1\end{bmatrix};\begin{bmatrix}2\\0\\1\end{bmatrix}\underset{[120]}{\rightarrow}\begin{bmatrix}0\\1\\2\end{bmatrix}.$$

Other inverters are also possible, for instance inverter [0 0 2]. It should be clear that these inverters are not reversible.

4-Valued Inverters

It is now fairly easy to extend the reversible inverter approach to n-valued inverters. As an example 4-valued reversible inverters will be demonstrated. There are 4! = 24 4-valued reversible inverters. Again there are self reversing inverters such as [1 0 3 2], [2 3 0 1] and [3 2 1 0]. The identity inverter [0 1 2 3] is also self reversible.

One universal inverter is [1 3 0 2]. Applying the inverter [1 3 0 2] four times is shown in the following:

$$\begin{bmatrix}0\\1\\2\\3\end{bmatrix} \underset{[1302]}{\rightarrow} \begin{bmatrix}1\\3\\0\\2\end{bmatrix};\begin{bmatrix}1\\3\\0\\2\end{bmatrix} \underset{[1302]}{\rightarrow} \begin{bmatrix}3\\2\\1\\0\end{bmatrix};\begin{bmatrix}3\\2\\1\\0\end{bmatrix} \underset{[1302]}{\rightarrow} \begin{bmatrix}2\\0\\3\\1\end{bmatrix};\begin{bmatrix}2\\0\\3\\1\end{bmatrix} \underset{[1302]}{\rightarrow} \begin{bmatrix}0\\1\\2\\3\end{bmatrix}.$$

One can see also clearly why the inverter may be called universal. By applying the inverter one or more times one can change any 4-valued state into any other 4-valued state.

The reversing inverter of [1 3 0 2] is [2 0 3 1]. One can easily check that by writing out the transformation.

N-valued inverters

One may apply the same methods to n-valued inverters. There are n! reversible n-valued inverters.

About Ternarylogic LLC: Ternarylogic LLC is a New Jersey based R&D company. Its mission is to create novel MVL technology. The company owns a portfolio of inventions related to scramblers/descramblers, sequence generators and sequence detectors, sequence correlators, gates and inverters based circuitry, non-binary multipliers, latches and other non-volatile memory elements, optical disk applications and MC-DSSS technology.